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Multi-frequency sensitivity analysis of 3D models utilizing impedance boundary condition with scalar magnetic potential

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***Abstract* –** In this work the inverse problem solution with iterative Gauss-Newton algorithm and Truncated Singular Value Decomposition (TSVD) is shown. For the goal function a norm l_2 was chosen. To solve the inverse problem, which consists in the identification of conductivity distribution in 3D model, the multi-frequency sensitivity analysis has been applied. The correctness of sensitivity calculation has been proved utilizing three different methods, namely the Tellegen's method of adjoint model, differentiation of stiffness and mass matrix, as well as sensitivity approximation by means of difference quotient. Regarding the effectiveness of those methods, the first one is preferred because of shortest computational time.

Considered 3D model

For 3D eddy current analysis in frequency domain the scalar magnetic potential φ with the impedance boundary condition has been used. In this case the magnetic field can be described by Poisson equation:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=\nabla\cdot\boldsymbol{T}_0, \qquad (1)$$

where the quantity $\boldsymbol{T}_0$ is the known electric vector potential [1], which exists only in the space of excitation coil. The interaction between probe and conductive plate has been modeled by means of impedance boundary condition [1,2]. Hence, the relation between tangential components of field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ on the surface of conducting material is given by

$$\mathbf{1_n}\times\boldsymbol{E}=Z_S\,\mathbf{1_n}\times(\mathbf{1_n}\times\boldsymbol{H}), \qquad (2)$$

where Z_s designates characteristic impedance of conductor.
In described model the thickness of excitation and measurement coil was the same g = 1,25 mm. The height of measurement coil was equal h_m = 2,5 mm, whereas excitation coil amounted h_e = 5 mm. The same value has been assumed for a side of quadrilateral base of absolute probe a = 7,5 mm. The distribution of excitation current density can be described using only one z-component of vector $\boldsymbol{T}$ [1].

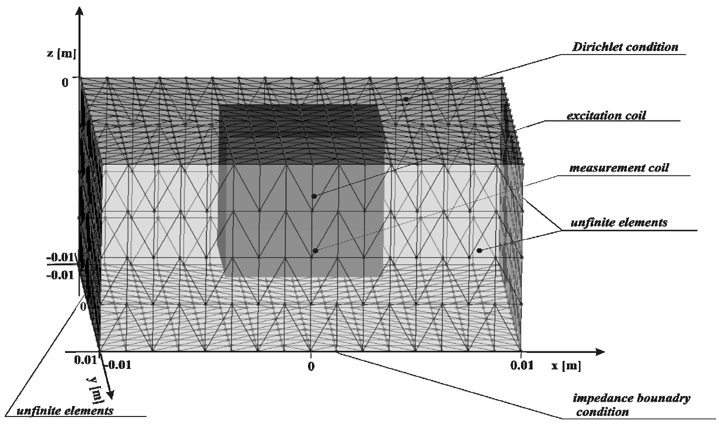

Fig.1 Square model of coil over conductive plate

Sensitivity analysis

The sensitivity analysis supplies gradient information for Gauss-Newton algorithm. From this reason the effectiveness of sensitivity module is very important for the reconstruction algorithm. Considering two methods of sensitivity evaluation, the adjoint model of Tellegen [2,3,5] and differentiation of stiffness/mass matrix [2,3], the first one provides sensitivity of induced voltage versus conductivity of all finite elements in one cycle. New cycle is necessary for other position of measurement coil. The second, differentiation method, requires additional loop to calculate sensitivity value for each element.
To obtain the sensitivity information utilizing the matrix differentiation method one should solve additional, so called incremental model:

$$\left(\left[\boldsymbol{H}_{\mathrm{P}}\right]+\left[\boldsymbol{H}_{\mathrm{SIBC}}\right]\right)\frac{\partial}{\partial\gamma^{e}}\left[\boldsymbol{\varphi}\right]=-\frac{\partial}{\partial\gamma^{e}}\left[\boldsymbol{H}_{\mathrm{SIBC}}\right]\left[\boldsymbol{\varphi}\right],\ e=1,2...E, \tag{3}$$

where $[\boldsymbol{H}_{\mathrm{P}}]$ is the global stiffness and mass matrix for Poisson equation, whereas $[\boldsymbol{H}_{\mathrm{SIBC}}]$ indicates the matrix of impedance boundary condition.
Using Tellegens method the solution of original and adjoint model is necessary. Both models differ only with boundary condition, excitation currents and material properties [2,3]. Then, the sensitivity of coil voltage versus conductivity is given by sensitivity equation

$$\frac{\partial U}{\partial\gamma^{e}}=\frac{\alpha}{2\left(\gamma^{e}\right)^{2}}\int_{S^{e}}\left(\frac{\partial\varphi_{1}}{\partial x}\frac{\partial\varphi_{2}}{\partial x}+\frac{\partial\varphi_{1}}{\partial y}\frac{\partial\varphi_{2}}{\partial y}\right)dS^{e},$$
$$\alpha=\sqrt{\mathrm{j}\omega\mu\gamma^{e}}=\left(1+\mathrm{j}\right)/\delta^{e},\delta^{e}=\sqrt{2/\omega\mu\gamma^{e}},e=1,2,...E, \tag{4}$$

where ω is the pulsation of excitation current, γ^e , μ^e indicates conductivity and permeability in finite element e and δ^e defines the equivalent penetration depth.
Comparing results of sensitivity calculation with both methods, the high compatibility level has been affirmed.

Conlusions

We propose simple algorithm for solution of 3D inverse problems of conductivity estimation basing on scalar magnetic potential and impedance boundary condition. For the solution of current distribution inside conductors the full 3D-formulation with four unknows at a node is necessary. The proposed methods of sensitivity evaluation will be applicable also in this case.

References

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